Using Item Response Theory To Rate (Not Only) Programmers

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Objectivity of ratings

Common questions

. . . (not only) when organizing a programming contest:

- is the evaluation method objective (enough)?
- is the task difficulty appropriate?
  (and what does "appropriate" mean?)
- will the best ones really win?

A usual goal of a contest

ranking = arranging the contestant into a linear order
based on their skill levels

. . . and a common tool

rating = a numeric approximation of the skill level

State of the art

Bayesian rating systems

- Contestant's performance:
  a normally distributed random variable
- Rating: its estimated mean
- An incremental approach:
  current estimate + new result $\longrightarrow$ new estimate

Examples

Elo	Elő, 1978.
Glicko	Glickman, 1999.
TrueSkill	Herbrich, Graepel (Microsoft) 2006.
TopCoder	TopCoder Inc. 2002–2008.

Disadvantages of existing rating systems

- They can not take task difficulty into account.
- They can only predict relative performance.
- Models usually ignore that performance may change in time.
- Low attack resistance.

What is IRT?

Setting:

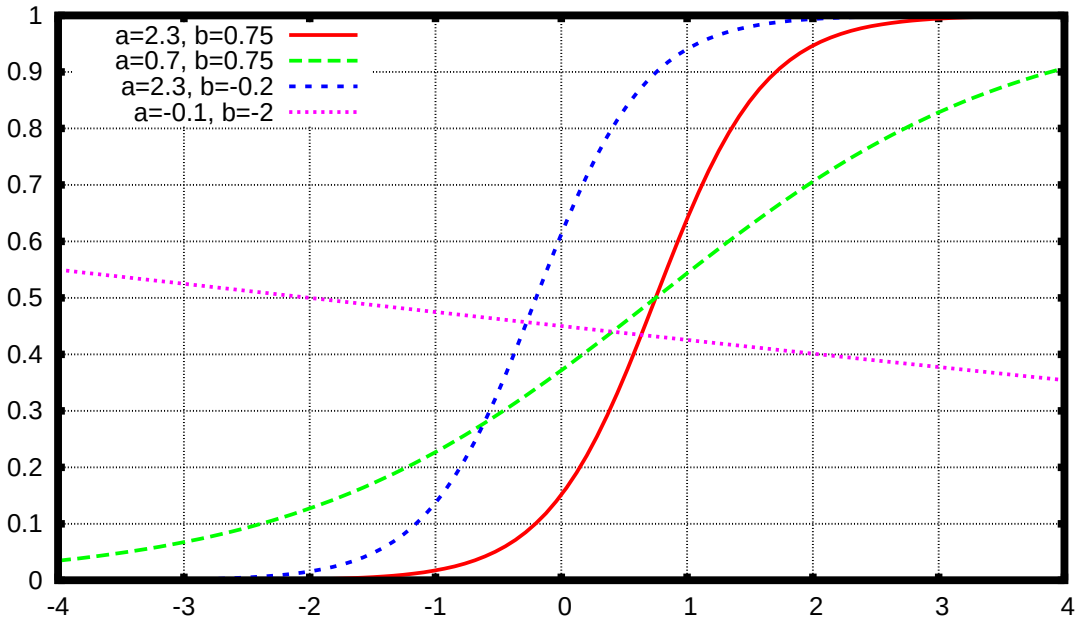
- people have latent abilities (e.g., problem solving)
- latent = we can not measure them directly
- goal of a testing theory: estimate them using tests

Item Response Theory: a modern testing theory
(basics developed in '50s and '60s, only becomes used in '80s)

Modelling task difficulty

Item Response Theory: 2-parameter logistic model

$$Pr(\theta, a, b) = \frac{1}{1+e^{-a(\theta-b)}}$$



Good enough for "binary" programming tasks,
and we managed to modify it for some types of partial scores.

Basic principle of use

To estimate the parameters (task difficulty, contestant skill) we may use the maximum likelihood estimate.

A simple example:
We know task parameters a_i, b_i, a response pattern s_i,
we are estimating ability θ.

$$L(\theta) = \prod_{i=1}^{n} Pr(\theta, a_i, b_i)^{s_i} \cdot (1 - Pr(\theta, a_i, b_i))^{1-s_i}$$

Fisher information

How to measure information?

Intuition: the reciprocal of the precision of the estimate

Formally: expected variance of the score
(score = partial derivative of the log-likelihood function)

$$\mathcal{I}(\theta) = E\left(\frac{\partial}{\partial\theta}\ln L(\theta, x)\right)^2 \tag{1}$$

Importance for us

We are able to compute *the amount of information* the results of a given *contest* give us about a *contestant's skill estimate*.

Overview

Some results of our research:

- The model fits real life data.
- It can be used to argue about task difficulty.
- Solving time in contests tends to have a log-normal distribution.
- Models can be compared via the predictions they make.
- Our model makes good predictions.

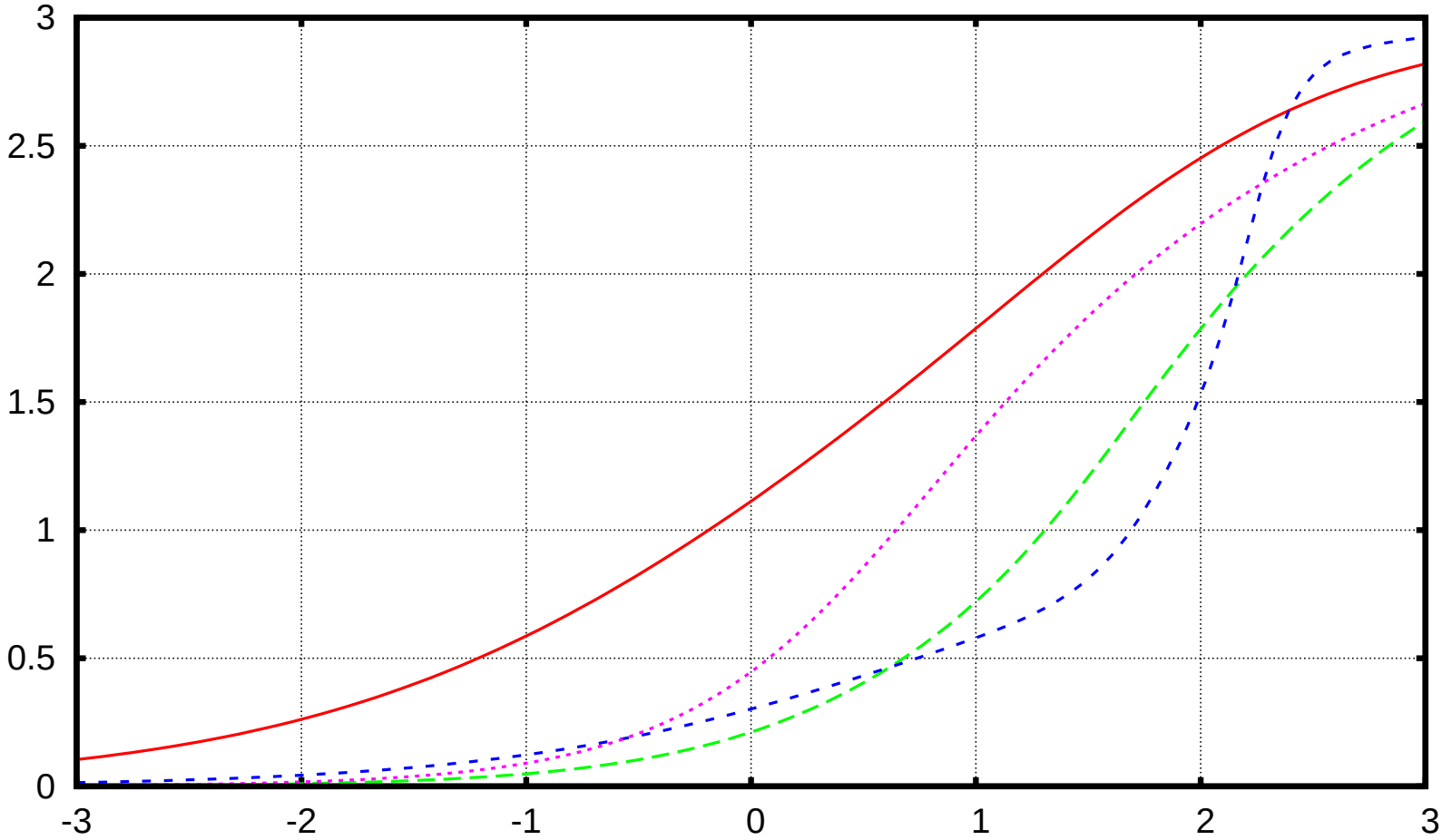
Sanity check

Corellation coefficients between the actual data
and the best fitting model
(Slovak programming competitions: 300 contestants, 88 tasks)

ρ range	year 2006/07	year 2007/08
$[0.99, 1.00]$	9	12
$[0.98, 0.99)$	6	5
$[0.95, 0.98)$	9	9
$[0.90, 0.95)$	7	7
$[0.75, 0.90)$	12	8
$[0.00, 0.75)$	1	2
$[-1.00, 0.00)$	0	1

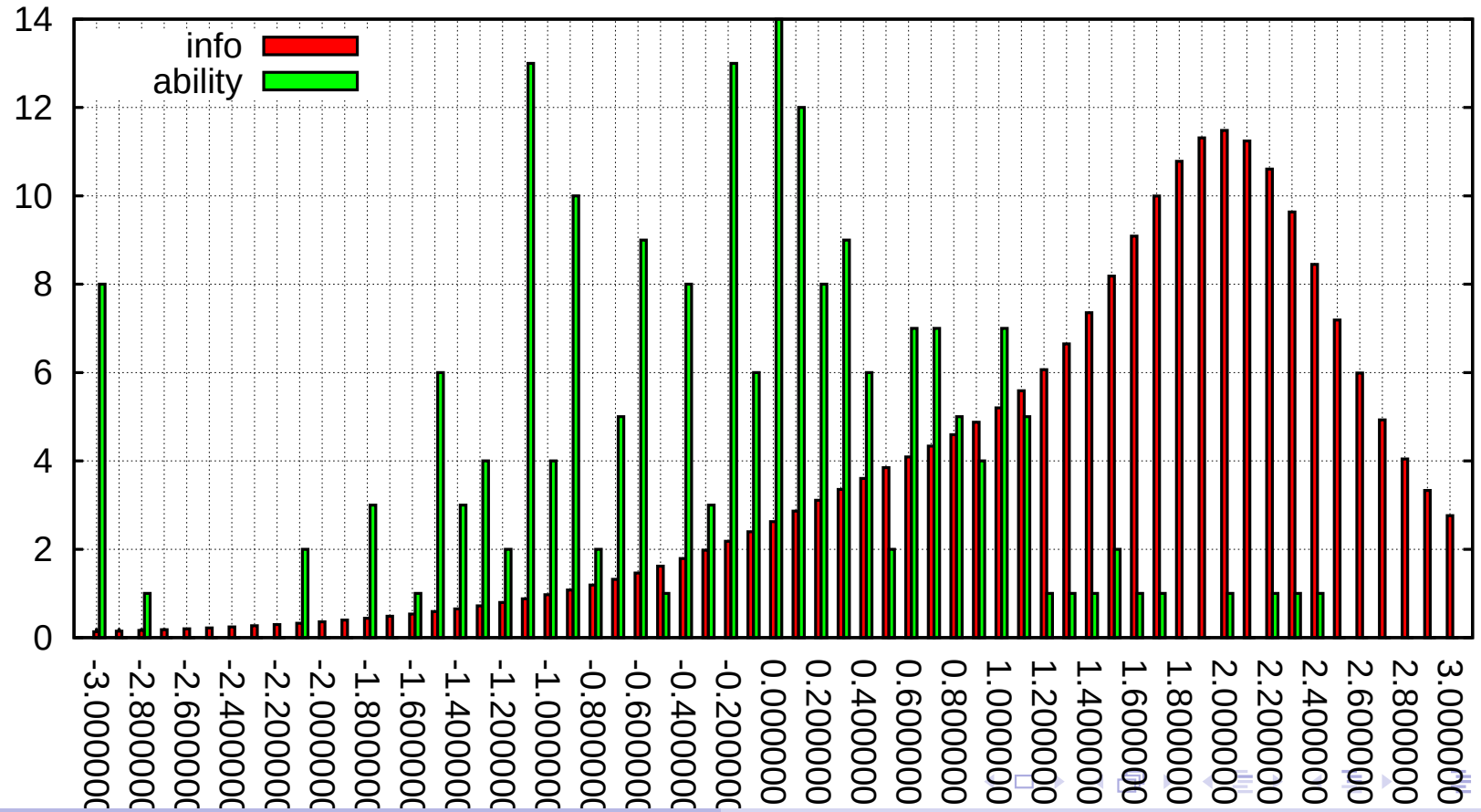
Regional round tasks

Four tasks of different difficulty: skill vs. expected score



Arguing about task difficulty

- skill estimate distribution (same data set)
- test information function for the regional round



Comparing predictions

Main observation

- You can not compare rating systems directly.
- Compare pairs (rating system, prediction algorithm)!

What can we predict?

Tournament advancement:
We have N contestants and a tournament round, K best advance.
For each contestant, predict the advancement probability.

How to evaluate it?

Compare their likelihood given the actual result:

$$Q(p_1, \ldots, p_N) = \sum_{i=1}^{N} s_i \log p_i + \sum_{i=1}^{n} (1 - s_i) \log(1 - p_i)$$

Advantages of IRT based ratings

Bayesian rating systems

. . . can only predict relative performance – we can estimate the probability that contestant A beats contestant B.

IRT-based rating system

. . . can predict absolute performance – we can compute the expected number of tasks A will solve.

(or generalize: probability that A solves x tasks,
expected score A gets, etc.)

Example: TopCoder Open 2008

We tweaked our model to approximately match the TopCoder competition. Many difficulties:

- We competely ignore challenges.
- We have to take a weird scoring function into account.

Still, we got quite lovely predictions:

- for round 1 a slightly better prediction
- for round 2 a slightly worse prediction
- for round 1: we predicted 872.944 advancers, reality: 864/900
- for round 2: we predicted 300/300 advancers, happened

Thanks for your attention!

Questions, comments?